FLOW DENSITY OF ADIABATIC EVAPORATING-LIQUID FLOWS

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A description is given of a semiempirical method of determining the flow density of nonequilibrium adiabatic flows of evaporating liquids based on the vortex theory of vapor-phase generation, which establishes a relation between the superheating of the liquid and its flow velocity. It is shown that, at subcritical flow rates, evaporation in an adiabatic liquid flow becomes impossible, and a method of determining the critical flow parameters is given.

When a thermally insulated liquid flows into a space where the pressure is lower than the saturation pressure at the initial temperature of the liquid, separation of a vapor phase may take place in the flow. This effect has been called adiabatic vaporization. In the literature, for example, in [1], an isentropic thermodynamic-equilibrium flow of homogeneous vapor-liquid mixture is assumed. However, this assumption is justified only when the initial vapor content x_{in} is sufficiently large, which is possible, for example, in the presence of preliminary throttling. If, however, the initial vapor content does not exceed 2%, the flow will be essentially nonequilibrium and assuming an isentropic thermodynamic-equilibrium homogeneous flow leads to a deviation of up to 450% from the experimental values of the two-phase flow density [2].

There is as yet no general theory of nonequilibrium adiabatic flows of evaporating liquids. In this paper, we attempt to find an approach to the general solution of the problem.

Experimental investigations of adiabatic vaporization show that the appearance of a vapor phase is accompanied by superheating of the liquid and takes place at pressures in the flow considerably exceeding the pressure at which the liquid loses stability when at rest, i.e., the superheating attainable in a flow is much less than in the quiescent liquid.

We determine the superheating of the flowing liquid, assuming that the vapor phase is generated in the eddies produced by turbulence. We note that in all known cases of adiabatic vaporization the Reynolds numbers of the flow are several orders greater than the Reynolds numbers at which the transition from turbulent to laminar flow takes place, and such flows are rotational.

There has been a series of studies relating to the investigation of the vortex structure of turbulent flows. The most interesting from our viewpoint is [3], in which it was experimentally confirmed that turbulence is the initiator of vaporization.

We examine a simplified plane-vortex model, assuming that at the moment of vortex formation the radial velocity distribution satisfies the condition of constant circulation. At the center of such a vortex there must be a zone of reduced pressure, where, in our model, the pressure at which the liquid state loses stability is attained and a vapor phase is generated. At the moment of generation of the vapor phase the vortex consists of an annular layer of liquid and a vapor core rotating about a common center.

We write the differential equation of motion for the liquid layer

$$\frac{1}{\rho'}\frac{dp}{dr}=\frac{u^2}{r},$$

where ρ' is the density of the liquid, constant along the radius r; and u is the circulation velocity.

Determining u from the condition of constant circulation $\Gamma = ur$, we have

$$dp=\frac{\rho'\Gamma^2}{r^3}\,dr. \qquad (1)$$

Integration of Eq. (1) over the radius gives

$$p_2 - p_1 = \frac{\rho' \Gamma^2}{2}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right). \tag{2}$$

In (2) the pressures p_1 and p_2 are determined within the liquid ring enclosed between two surfaces layers of liquid of radii r_1 and r_2. The pressure at the vapor-liquid interface, taking the surface tension σ into account, is

$$p_{V1} = p_1 - \frac{2\sigma}{r_1}, \tag{3}$$

and the pressure at the edge of the liquid vortex on the mainstream side

$$p_{V2} = p_2 + \frac{2\sigma}{r_2}. \tag{4}$$

Using (3) and (4), we reduce Eq. (2) to the form

$$p_{V2} - p_{V1} = \frac{\rho' \Gamma^2}{2}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) + 2\sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \tag{5}$$

The pressure p_{V2} determines the superheating of the liquid and corresponds to the pressure of vapor generation in the liquid flow.

The pressure p_{V1}, corresponding to loss of stability in the liquid at rest, is calculated from the formula [4]

$$p_{V1} = \frac{p_s v_s''}{v_s'} \exp\left(-\frac{L}{RT}\right),$$

where p_S, T, v_S'', and v_S' are the pressure, temperature, and specific volumes of the saturated vapor and liquid; L is the heat of vaporization; and R is the gas constant.

To determine r_1, we write the differential equation of motion for the vapor core

$$\frac{1}{\rho''}\frac{dp}{dr} = \frac{u^2}{r}. \tag{6}$$

Assuming an exponential law of radial density distribution

$$\rho'' = \rho_s'' \exp\left(1 - \frac{r_1}{r}\right),$$

where ρ_S'' is the density of the saturated vapor at the interface with the liquid, from (6) we have

$$dp = \frac{\rho_s'' \Gamma^2}{r^3} \exp\left(1 - \frac{r_1}{r}\right) dr.$$

Integration of this expression from r_0 to r_1 gives

$$p_{V1} - p_0 = \frac{\rho_s'' \Gamma^2}{r_1^2}\left[2 - \left(1 + \frac{r_1}{r_0}\right) \exp\left(1 - \frac{r_1}{r_0}\right)\right]. \tag{7}$$

As $r_0 \to 0$ the limit of the subtrahend is equal to zero. Thus, finally, as $p_0 \to 0$ from (7) we obtain

$$r_1 = \Gamma \sqrt{\frac{2\rho_s''}{p_{V1}}}. \tag{8}$$

The existence of a vortex is possible if $\chi = r_1/r_2 < 1$. Using (8) and assuming that $r_2 = \lambda/2$, we have

$$\chi = \frac{2\Gamma}{\lambda}\sqrt{\frac{2\rho_s''}{p_{V1}}}. \tag{9}$$

Substitution of (8) into (5) and certain transformations lead to the following expression for the vaporization pressure and the corresponding superheating in the liquid flow:

$$p_{V2} = p_{V1}\left[1 + \frac{(1-\chi^2)}{4}\frac{\rho'}{\rho_s''}\right] + \frac{2\sigma(1+\chi)}{\Gamma}\sqrt{\frac{p_{V1}}{2\rho_s''}}. \tag{10}$$

In the first approximation, the parameters ρ', ρ_g'', σ, and also p_{V1} are determined at the initial temperature of the liquid T_{in}.

In a turbulent flow, vortices of different sizes are possible and, in particular, vortices with scales λ_0 and $\lambda > \lambda_0$, whose circulation velocities are, respectively, w_0 and $u = kw$, where w is the flow-rate velocity of the pure liquid at its initial density ρ'. Using the law of conservation of total moment of momentum in the form [5]

$$w_i^2 \lambda_i^5 = \text{const}, \tag{11}$$

we establish a relation between the circulations of vortices of different scales. For the characteristic values of the smallest-scale turbulence we obtain

$$\lambda_0 \sim \frac{d}{\mathrm{Re}^{3/4}}, \tag{12}$$

$$w_0 \sim \frac{u}{\mathrm{Re}^{1/4}}, \tag{13}$$

where Re = wd/v; d is the diameter of the channel flow cross section.

Substituting (12) and (13) in the form of an equality into (11), we have

$$\lambda = \frac{nd}{\mathrm{Re}^{0.85}}. \tag{14}$$

The circulation is given by

$$2\Gamma = kw\lambda. \tag{15}$$

The coefficients k and n in (14) and (15) are always selected so that the best possible agreement with the experimental data is obtained.

A comprehensive analysis of Eqs. (9), (10), (14) and (15) shows that p_{V2} is a nonlinear function of w. Thus, the vortex theory of vapor phase generation establishes an important theoretical relation between the superheating of the liquid and the flow velocity.

There is certain velocity of the vortex flow at which the superheating becomes zero and adiabatic evaporation impossible. We describe this velocity of the turbulent flow and the corresponding pressure drop as critical.

In the absence of vortices, for example, in the presence of laminar flow, $\Gamma = 0$ and $pV_2 = \infty$, i.e., if there are no vortices in the liquid, then, in accordance with the theory, the generation of a vapor phase is impossible.

We have developed a semiempirical method of calculating the flow-rate density of nonequilibrium adiabatic flows of evaporating liquids based on the application of the vortex theory of vapor-phase generation.

The essence of the method consists in the following. If the initial parameters of the liquid, the dimensions of the flow cross section, and the empirical coefficients k and n are known, using the system of equations (14), (15), (9) and (10) we establish a first analytic relation $w = f(p_{V2})$. The second analytic relation between w and p_{V2} is obtained from the system of equations of conservation of mass, momentum, and energy and from the expressions for the plase slip ε and the true vapor content φ. One control section is located in the pure liquid flow, the second in the section determining the capacity of the vapor-generating element.

In practice, it is more convenient to employ a graphical construction. Assigning a series of values of w, from the system of equations (14), (15), (9) and (10) we find a series of values of p_{V2} and construct the first graphical relation $w = f(p_{V2})$. Assigning a series of values of p_{V2}, using the system of conservation equations we compute a

series of values of the flow-rate velocity w and construct the second graphical relation $w = f(p_{V_2})$. The quantities ρ', ρ'', L and i' entering into the conservation equations are determined from thermodynamic tables at the pressure p_{V_2}. The point of intersection of the first and second graphical relations uniquely determines the velocity w and the flow-rate density $\rho'w$. All the properties of real flows are reflected by the empirical coefficients k and n.

The empirical coefficients k and n are determined from experimental flow-rate characteristics. By successive approximations we find the values of p_{V_2}, at which the values of ρ', ρ'', L and i' satisfy the conservation equations at the experimentally known velocity w. Then the values of p_{V_2} obtained are related with the same velocity w through the coefficients k and n using the system of equations (14), (15), (9) and (10). The following empirical relation has been established between the average values k_{av} and n_{av}:

$$k_{av} = n_{av}^2 \mathrm{Re}^{-0.45}.$$

The flow-rate characteristics for cylindrical channels carrying water calculated from the vortex theory are presented in Fig. 1 [2]. The greatest discrepancy with experiment does not exceed 20%. The following average values of the empirical coefficients were employed: $l/d = 1.6$; $k_{av} = 0.48$, $n_{av} = 16.3$; $l/d = 5.2$, $k_{av} = 0.837$, $n_{av} = 19.6$; $l/d = 100$ $k_{av} = 1.73$, $n_{av} = 26.8$.

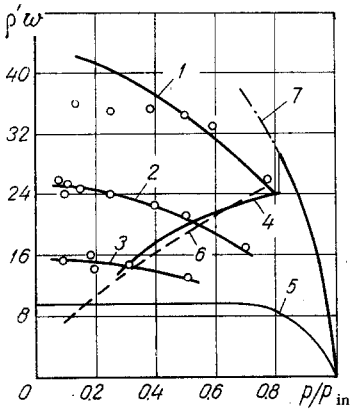

Fig. 1. Flow-rate characteristics of cylindrical channels for water (p_{in} = $= 19.5 \cdot 10^5$ N/m^2; $\rho'w$, kg/cm^2 · sec): 1, 2, 3) according to the vortex theory; 4) line of critical conterpressures; 5) thermodynamic-equilibrium homogeneous flow; 6) equilibrium stratified flow; 7) hydraulic flow ($1 - l/d = 1.6$; 2–5.2; 4–100).

The characteristics for circular convergent nozzles and water [2] at $k_{av} = 0.561$ and $n_{av} = 19.3$, calculated from the vortex theory, are presented in Fig. 2. In this case the maximum discrepancy does not exceed 3%.

In Fig. 3 the experimental [1, 6] characteristics for Laval nozzles and water are compared with those calculated from the vortex theory using the following empirical coefficients: for $p_{in} = 1.5 \cdot 10^6$ N/m^2 $k_{av} = 0.384$ $n_{av} = 17.4$; $p_{in} = 3.5 \cdot 10^6$ N/m^2 $k_{av} = 0.31$, $n_{av} = 17$; $p_{in} = 5.5 \cdot 10^6$ N/m^2 $k_{av} = 0.271$, $n_{av} = 16.9$; $p_{in} = 7 \cdot 10^6$ N/m^2 $k_{av} = 0.294$, $n_{ac} = 18.3$ (p_{in} is the pressure of the water upstream from the nozzle). The calculations were made for a pressure in the nozzle throat equal to p_{V_2}. The error in the calculations based on the above values of k_{av} and n_{av} does not exceed 2% for selected experimental points.

In all cases, the phase slip and the true vapor content were determined from the formulas

$$\varepsilon = \left(\frac{\rho'}{\rho''}\right)^{1/2},$$

$$\varphi = \frac{1}{1 + \frac{1-x}{x}\left(\frac{\rho''}{\rho'}\right)^{1/2}}.$$

The vortex theory also makes it possible to compute the critical velocity and critical pressure drops, at which adiabatic vaporization in the flow becomes impossible.

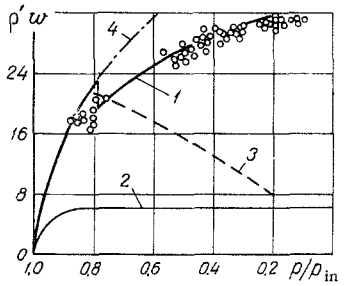

Fig. 2. Flow-rate characteristics of circular convergent nozzles for water ($p_{in} = 14.5 \cdot 10^5$ N/m^2): 1) according to the vortex theory; 2) thermodynamic-equilibrium homogeneous flow; 3) equilibrium stratified flow; 4) hydraulic flow.

The order of calculation is as follows. First, the flow-rate characteristics are computed. Then, at each point of the characteristic the differences ($p_{V_2} - p$) are calculated and the graphical relations ($p_{V_2} - p$) = f(Re) and $p = f$(Re) constructed. The point of intersection of the first relation and the axis of abscissas, where $p_{V_2} = p$, determines the critical number Re_{cr} and the critical velocity w_{cr}, at which there is no superheating in the liquid and evaporation ceases. The second relation is used to determine the critical pressure p_{cr}. At velocities less than w_{cr} we get purely hydraulic flow and the flow-rate density changes abruptly.

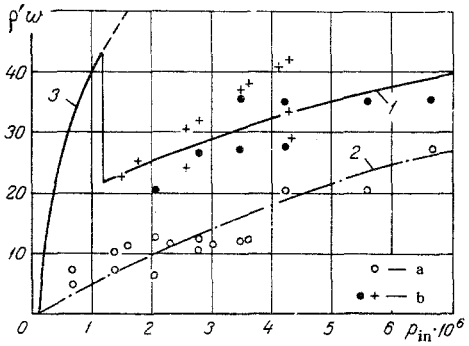

Fig. 3. Flow-rate characteristics for a Laval nozzle and water (p_{in}, N/m^2): 1) according to the vortex theory; 2) thermodynamic-equilibrium homogeneous flow; 3) hydraulic flow; a) $x_{in} > 5\%$ [1]; b) $x_{in} < 2\%$ [1, 6].

The calculated values of the critical parameters have been plotted in Figs. 1–3.

For cylindrical channels and circular convergent nozzles, the value of p_{cr} corresponds to the counterpressure in the outlet section p. For Laval nozzles p_{cr} corresponds to the least initial pressure p_{in} of the liquid upstream from the nozzle. We note that the critical velocities of water flows obtained from calculations based on the vortex theory lie in the range 20–30 m/sec, which is consistent with the experimental data of [3].

NOTATION

p is the pressure; ρ is the density; v is the specific volume; T is the temperature; R is the gas constant; L is the heat of vaporization; σ is the surface tension; ν is the kinematic viscosity; i is the enthalpy; r is the radius; d is the diameter; λ is the scale of turbulence; l is the length; f is the cross-sectional area; w is the velocity; u is the circulation velocity; $\varepsilon = w''/w'$ is the phase slip; Γ is the circulation; x is the mass vapor content; $\varphi = f''/f$ is the true vapor content. Subscripts: (') are liquid parameters; (") vapor parameters; (in) initial parameters; s is a saturation parameter; V is a vapor-generation parameter; cr is a critical parameter; av is an average parameter.

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